Numeracy Nugget #10: The WoRm Formula

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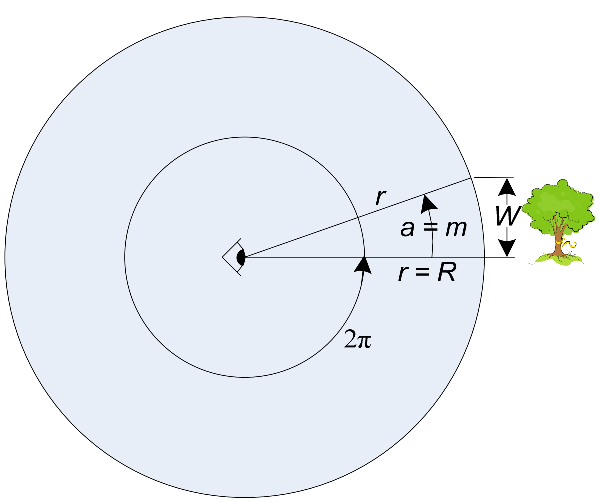
There is a very simple technique for measuring width/height of an object when you know the distance to it, and conversely, for measuring the distance to something whose lateral dimension you know. It's neat stuff that frequently comes in handy for hunters, campers, golfers, sailors, hikers, … when you know the method.

The method is based on the definition of natural angle and is used a lot in the military, primarily in artillery and naval gunnery. Natural angle? Well, we all know a degree is the angle that a piece of a circle's circumference makes (subtends) when you divide the circumference into 360 equal parts. Although we're all used to a degree and the 360 equal parts, the 360 is totally arbitrary and probably comes from ancient astronomy when that may have been the best guess for the number of days in a year. You could have just as well divided the circumference into 400 equal parts or any other convenient number. (BTW, dividing it into 400 parts makes each part subtend an angle called the 'grad' or 'gradian' so that a right angle is 100 gradians.)

All these kinds of angles are really manmade. The natural angle of a full circle comes about when you divide the circumference of any circle by its radius r. From high school we all know that the circumference $C = 2\pi r$ where it turns out $\pi = 3.14159\ldots$. This gives $C/r = 2\pi$ 'something' for the total angle of a circle instead of 360 degrees or 400 grads. That 'something' is called radians, so that there are $2\pi = 6.2832\ldots$ radians in a full circle and a right angle then has the natural angle $2\pi/4$ or $\pi/2$ radians. Radians are a natural angle because our universe is so constructed that no matter where you go, the ratio of any circle's circumference to its diameter ($2r$) is always π – this makes the number π one of the special constants that define our particular universe.

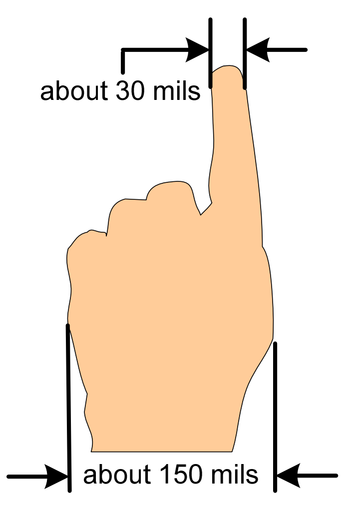


Well then what is exactly one radian? It's the angle subtended by the arc of the circumference that is as long as the radius, simply r/r = 1. Since 2π radians = 360 degrees, one radian is $360/2\pi$ = 57.2958 … degrees. So what's all this got to do with measuring distance etc? A little patience, and you'll see it all come together.

Say that we examine the picture of taking a length W that is much smaller than radius r. If we place this W on the circle's circumference, then the picture is as shown in the figure above (here *W* is exaggerated). In radians the angle *a* that *W* subtends at the center of the circle is $a = W/r$, it's that simple. Now we're almost done. If we solve this angle formula for width *W*, we get $W = a*r$. Solving it for radius, or let's call it range, we get $r = W/a$. Take $W = a*r$ and multiply the right side by 1 = 1000/1000, this doesn't change things at all. $W = a*r*(1000/1000)$ but let's rearrange that a bit so that $W = (m*1000)*(r/1000)$. Notice that now the angle part is still correct if *a* is expressed in *m* milliradians (thousandths of radians), or $m = a/1000$. And the range part is correct if $r/1000$ is expressed in *R* **kilounits** (thousands of units) of distance like kilofeet, so that $r = 1000*R$. From here on let's shorten milliradians to simply **mils**.

At this point you can forget all about the stuff we just covered. Our basic relationship comes down to $W = m*R$ without changing anything except scaling the angle now expressed in *m* mils and scaling the range to, say, kilofeet. Again we have $R = W/m$. Now let's just write the expression $W/(R*m)$ stated '**W**idth **o**ver **R**ange times **m**ils' or just remember **WoRm**. That little expression automatically tells you the formula for computing any of the three parameters that make up the natural angle formula in terms of the remaining two parameters. Simply cover up the parameter you want to calculate and bingo you have it. Bottom line again, just remember **W/(R*m)** pronounced 'worm' as in the **WoRm Formula**. If you want *W*, then covering it leaves you with $R*m$; if you want *R*, then covering that leaves you with W/m; and if you (seldom) want *m*, then covering it you have W/R.

So how do you use the WoRm formula in a practical situation. Well, first you have to be able to get a handy measure of the angle *m* that some distant width *W* will subtend at range *R*. If you're in the military, or want to spend the money, you buy binoculars with a 'mil reticle' in one of the lenses. This lets you just look at the object and read the mils it subtends or how wide (or tall) it appears in mils. Suppose you don't have such binoculars; no problem if your handy angle measurement device is your very own hand.



In Artillery School at Ft. Sill, Oklahoma all young lieutenants immediately 'calibrate their hands' so that they can become expert forward observers in any situation. Almost every other building on the base has on it a big ruler with tick marks and every fifth tick mark is numbered. There is also a stripe painted on the pavement that is one thousand tick marks from the wall with the ruler. If the tick marks are in inches, then the stripe is 1,000 inches or 83 feet 4 inches from the wall. Alternatively, one inch tick marks will subtend 10 mils if your toes are 100 inches = 8 feet 4 inches from the ruler (use this at home). Either calibration set up will let you get the angles of your hand close enough for government work.

To calibrate your hand, you just put your toes on the line and point your arm at full extension to the scale. Now with your fingers extended one at a time, read off how many mils wide each of them is. Usually a person's index, middle, and ring fingers are about 25 to 30 mils wide with the pinky being about 20 to 25 mils wide. Don't worry about measuring these finger widths too precisely, the objective here is to wind up with a good ball park estimator. You should also measure the width of your fist and your hand with thumb and little finger at full extension. Now memorize your finger, fist, and full extension numbers and you're in business. For example, my three fingers are about 30 mils wide, pinky is 25, fist is 150 mils, and full extension is 300 mils wide.

We'll conclude with a couple of examples and then you're off to having some fun amazing family and friends. All these examples use my hand measurements and your ability to do simple mental arithmetic (or use a calculator).

Say, you see a person standing some distance away and you want to know what that distance is. Pointing to him you see that he's about ¼ of your finger width or 7.5 mils tall. A person is usually somewhere between 5.5 and 6 feet tall. You decide 6 feet and from the WoRm Formula your problem to compute *R* becomes $W/m = 6/7.5$ which you can do in your head by $(7.5 - 1.5)/7.5 = 1 - 1.5/7.5 = 1-1/5$ or approximately 0.80 (remember your division tables which tells you that $1/5 = 0.2$). So $R = 1000*0.80$ or approximately 800 feet. Remember *R* from the WoRm Formula is in kilounits of *W*, in this case feet.

Here's a more complex problem that becomes easy. Say, you're an 'Anthony Watts volunteer' surveying the nation's temperature monitoring stations located in all kinds of accessible and inaccessible locations. The station you're looking at is behind a big chain link fence so you can't get to it to measure its distance to a nearby black asphalt parking lot that may skew its temperature readings during sunny days. You know the white box containing the instrumentation is the standard 2 feet wide. You point to it and determine that the box subtends about half a finger width which makes $m = 15$ mils. First you need to find *R* which from the WoRm Formula is again $R = W/m = 2/15 = (1.5 + 0.5)/15 = 0.133$ or $1000*0.133 = 133$ feet (you could round this to 130 feet if you wish). With *R* in hand you again point to the distance between the white box and the asphalt and determine that it's three fingers wide or $m = 3*30 = 90$ mils. Now you're calculating the distance *W* knowing *R* and *m*. Again from the WoRm Formula you get $W = R*m$. Recall you're using *R* in kilounits or, in this case, $R = 133/1000 = 0.133$. This gives you $R*m = 0.133*90 = (0.1 + 0.033)*90 = 9 + 3 = 12$ feet. So you log that the temperature monitoring station sits 12 feet away from the asphalt parking lot, and you did all this from behind a chain link fence over a 130 feet from the inaccessible instrument.

Solution to NN9 Problem – Inflation Adjusted Appreciation. (First we'll restate the problem) The common formula for calculating inflation adjusted appreciation given by the media and almost all investment advisors is wrong. The common formula known by most people is that the adjusted rate of appreciation of an investment that appreciates *I*% over a period, say, one year during which the rate of inflation is *F*% is simply (*I-F*)%. This is a crude approximation that gets worse as the inflation rate increases. Derive the correct formula to use for one period. Hint: Find the buying power of an appreciated dollar that is spent for inflated goods to yield A the actual rate of appreciation. To check yourself assume that I = 15% and F = 10%. The common answer of $A = I - F$ gives 5% instead of the correct answer 4.55%.

Inflation takes a significantly bigger bite out of invested dollars whether you invest them under your mattress or in some more sophisticated asset. For extra credit derive the formula for N periods where I and F are the periodic rates defined above. Putting $N = 1$ should again give you A = 4.55%. And if you invest for N = 5 years with the annual rates I = 15% and F = 10%, then you should get 24.89% for the total appreciation instead of 27.63% which is the wrong answer using the common formula for annual return compounded over 5 years. Deriving the correct solution requires nothing beyond a bit of high school algebra and clear thinking. You will be amazed at how the error in the common formula grows for actual appreciation of multi-year investments.

Solution: Suppose today you pay \$$D$ for an amount C bushels of corn. After, say, one year the same amount C costs $(1+F)D$ to reflect the inflation rate F% over the same period. Now if you had invested D and received a return of I%, then the amount you would have after one year is $(1+I)D$ to spend on corn. The inflation adjusted appreciation of your investment is determined by how much corn you can buy after a year. Since future price of corn in, say, bushels/dollar will be $C/[(1+F)D]$, your $(1+I)D$ dollars buys

$$\frac{CD(1+I)}{D(1+F)} = C\left(\frac{1+I}{1+F}\right).$$

The real buying power of your invested amount increased by the additional A% of corn you can then buy. A is calculated from the standard percent appreciation formula

$$A = \frac{C\dfrac{(1+I)}{(1+F)} - C}{C} = \frac{I-F}{1+F}$$

Now assume that your investment pays I%/yr for N years and inflation is a constant F%/yr over the same interval. Then using the above value for A in the well-known N-period appreciation formula, we get the total real appreciation as

$$A_N = (1+A)^N - 1 = \left(1 + \frac{I-F}{1+F}\right)^N - 1 = \left(\frac{1+I}{1+F}\right)^N - 1$$

We'll leave you to modify these formulas to account for the tax rate of R% paid when the investment is cashed out. Now you know why using the common I-F doesn't give you the correct answer.

NN10 Problem – Early Train, Missed Taxi, Some Walking Required (This clever problem is from a Mensa quiz book for testing if you're a genius.) "A man hires a taxi to meet him at the railroad station at 3PM to take him to an appointment. He catches an earlier train and arrives at 2PM. He immediately decides to start walking and is picked up en route by the taxi driving to the station. He arrives 20 minutes early for his appointment. How long did he walk?"