TechTest2014

Merit Scholarship Examination
in the Sciences and Mathematics
given on 5 April 2014, and

sponsored by

The Sierra Economics and Science Foundation

Summary of rules and regulations for the exam:

1. Students will bring their own unmarked blue books to the exam; 3 to 5 blue books should suffice. On the cover of each blue book print your name, today's date '5 April 2014', the name of the exam 'TechTest2014', and the number/totalNumber of the blue book; e.g. the second of three blue books would be labeled 2/3.
2. Students will have four hours to take the test, 9am-1pm. Blue books not handed in by 1pm will not be graded.
3. There will be no passing or exchanging any kind of materials or information between test takers. If there is a problem, contact a proctor.
4. Graphing calculators are allowed and recommended.
5. Two, 8.5" X 11" 'cheat sheets' may be used under the following conditions: They are handwritten (both sides if desired) with your name clearly printed on each sheet.
6. No cell phones or other communication and/or information retrieval devices of any kind may be used at any time during the test.
7. Students may leave the test to go to the restroom, one at a time as permitted by the proctor.
8. Juniors will be allowed to take the test under the rules here stated, but will not be considered for scholarship awards. The eligibility requirements for Juniors remain the same, 3.0 min gpa, specified majors, etc.
9. The determination and announcement of the winners is final. No re-grading or petitioning of any kind is allowed once the winners are determined.
10. All students will do and submit only their own work.
11. Recommendation: Read all the problems first and note their point values (Ten problems, 100 points total). Since partial credit will be given, make a plan on which order and to what point of completion you wish to solve each problem, so that you maximize your score in the time allowed.

Problem 1: Trigonometric Equation (5 points)

Solve $\sin x + \cos x = 1$.

Problem 2: Differentiation (5 points)

Given $y = \dfrac{e^{ax}}{1-\sin^2 x}$, find $\dfrac{dy}{dx}$. (Hint: express y as a function of subfunctions of x and use the appropriate derivative formula for that expression to simplify your work.)

Problem 3: Integration (15 points)

It is easier to evaluate some definite integrals by using substitution to change the integration variable, and then, of course, its commensurate limits. Consider the requirement to compute $\int_{y_1}^{y_2} f(y)\,dy = \int_{\Phi(a)}^{\Phi(b)} f(y)\,dy$ where now $y = \Phi(t)$. We presume that a simpler integration problem would be to solve $\int_a^b g(t)\,dt$ such that $g(t) = f(y)\dfrac{dy}{dt} = f\left[\Phi(t)\right]\dfrac{dy}{dt}$, noticing that substituting the new integrand $g(t)$ into the integral would in effect 'cancel out' the dt, thereby computationally leaving the required dy of the original integral intact even though the new variable of integration is now t. The new limits a and b would be obtained from solving $y_1, y_2 = \Phi(t)$. This approach also lets us integrate mixed function integrals such as $\int_{y_1}^{y_2} h(t)\,dy$ where again $y = \Phi(t)$, which we now ask you to do in this problem. (Hint: find dy and substitute in the integral.)

Let $y = 3t^2 - t$, and calculate $z = \int_0^4 (4t+1)\,dy$.

Problem 4: Area of a Circle (10 points)

Derive the formula for the area of a circle of radius R given that its circumference $C = 2\pi R$.

Problem 5: Volume of a Right Cone (10 points)

Derive the formula for the volume of a right cone of base radius R and height H.

Problem 6: The Price of a Bond (10 points)

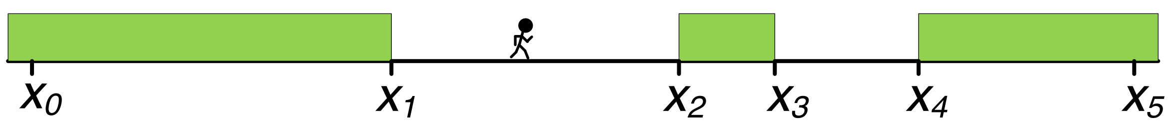
When issued, a long term Treasury bond that yields (over its life) an annual coupon amount of C is sold for price P. This says that at issuance the bond has an interest 'percent' yield of $I = C/P$. As interest rates go up and down, the market price of the bond will change so that its coupon amount reflects the current market's interest rate. A, 40%) If at a future time this interest rate becomes $I + \Delta I$, then what is the new market price of the bond? B, 40%) Express the fractional (percent) change in a bond's price in terms of the fractional change in the interest rate. C, 20%) If P = \$10K, C = \$500, and I changes from 5% to 6%, what is the percentage change in the market price of the bond?

Problem 7: Galileo climbs the Tower of Pisa (5 points)

Galileo is reputed to have climbed the leaning tower to settle once and for all Aristotle's claim that heavier objects fall to the ground faster. The experiment showed that, neglecting air resistance, both heavier and lighter masses were accelerated equally and hit the ground (almost) concurrently. Now Galileo did not have the benefit of Newton's laws, but you do. Prove that in a vacuum masses m and $10m$ have the same acceleration profile and hit the ground at the same time when attracted by the mass of the earth $m_E >> m$.

Problem 8: Where to look, when, and for how long (20 points)

Suppose you are charged to report on the whereabouts of a person who walks the same path every day at the same approximate time. As shown in the (not to scale) figure, her walk takes her back and forth on a circuit between two points, x_0 and x_5, over a straight path on which she is not visible to you the whole time because there are obstructing walls that will not let you see her from your chosen vantage point. She walks at a constant speed of V = 4,500 yards per hour. Her path is $x_5 - x_0$ = 1,500 yards long and defiladed by high walls as shown. She is known to walk at least one hour during her exercise session. Letting x_0 = 0, the coordinates of the other points along the path are x_1 = 500 yds, x_2 = 900 yds, x_3 = 950 yds, x_4 = 1,150 yds, x_5 = 1,500 yds.



Your objective is to ascertain and report her presence or absence as quickly as possible. When you arrive at your observation point, she is nowhere in sight. You note the time (t_0) and start scanning for her whereabouts. Since you cannot look concurrently at all points on the visible portions of her path, your scanning is made up of a sequence of glimpses that cover a more narrow focus and of a duration enough to ascertain with high likelihood that the walker is there or not. Describe a reasonable scanning policy (sequence of glimpses) which will allow you to meet your objective. In short, where will you look (focus your attention) when and for how long? (Hint: Your answer will be a written description with all additional assumptions clearly stated and appropriate numbers included as necessary.)

Problem 9: Rendezvous at Sea (10 points)

Two ships are steaming abeam, 50 miles apart, on parallel courses, and at the same speed of 15 knots. The ship A on the left is ordered to rendezvous with the ship B on the right as soon as possible. Ship B will maintain its course and speed. Ship A's maximum speed is 25 knots. A) What course and speed change will enable Ship A to comply with the minimum time to rendezvous requirement? B) What then is the course change and minimum time to rendezvous from the time that Ship A changes speed and course? (Hint: The speed of one knot equals one nautical mile per hour. 1 nautical mile = 1.15078 miles)

Problem 10: Packing Loss (10 points)

A large volume is divided into cubes with side length *D*. Into each cube is placed a sphere of maximum volume. Then between these spheres are placed smaller spheres of maximum volume. What fraction of the large volume is consumed by such placement of spheres? (This is known as the *packing density*. The residual fraction is known as *packing loss*.)

